Portfolios Theories and the Management of Customized Domestic Equity Funds: Pedagogy

Thomas A. Rhee California State University, Long Beach

Such inputs as securities expected return and variance-covariance matrix in constructing protfolios are forward looking quantities, which determine ex ante Efficient Portfolio Frontier. Many hedge funds and separate equity accounts managers use unique pricing models of their own. They in turn decide on the value of these inputs, which in the end influences the portfolios performance. This paper offers pedagogy as to how portfolios can be customized in varied investment environments and under differing clients' requirements. It uses some of the most popular interest rate and other equilibrium asset pricing models.

Today, many customer investment accounts are managed separately on an individual basis dedicated to the customer's specific need. The securities industry now derives fee incomes from managing individually tailored portfolios rather than from charging trade commissions. Many different forms of hedge funds have also emerged particularly targeting high net worth individuals. The industry also introduced what is known as wrap accounts, which are externally managed investment accounts, and advice based planning platforms, where customers are directed either to a particular mutual fund or to a fund of funds called the umbrella fund. What is common in these developments is that investors seem to prefer their own customized portfolios, which are managed privately, hopefully on an exclusive basis. Several factors may contribute to this phenomenon.

The first and foremost important reason is that any well performing public fund only draws more buyers and eventually the returns will suffer in the long run. On the

Rhee

contrary, private funds that perform well are not subjected to that market abuse. Neither hedge funds nor separate account portfolios are publicly traded. Therefore, investors find it attractive to have their portfolios customized and managed by professionals. In their minds, separately managed accounts offer a viable alternative to the conventional wisdom that investors need to reconfigure their mix between an index fund and the risk-free asset constantly, as the market condition changes. And it can be quite expensive. Separately managed accounts or hedge funds charge fees based only on the size of the asset, and not on the trading frequency. Most of the time there are no uncertainties about the account's transaction cost over the contract's duration. As a matter of fact, fees charged to separate accounts or even to the majority of hedge funds are no higher than the management fees embedded in any publicly traded index fund and the cost of frequently rebalancing the portfolio of an index fund and the risk-free asset.

Principles involved in managing separate accounts so as to meet the individual's risk-return requirement or even to meet the "absolute" return objectives by any hedge fund cannot be different from those followed in forming any optimal portfolios. In this paper some of these principles will be reviewed. Consequently, it is not claimed that the technology presented in this paper is new, but a pedagogy of forming various custom tailored portfolios by putting together some popular models of interest rates and equilibrium asset prices is offered. A high level discussion on developing a portfolio management system is also presented.

Why Individually Tailored Portfolios?

Capital market theory in modern finance proposes that investors can reach an optimal portfolio position based on their risk preference by mixing the market portfolio with the risk-free asset. Since no one really knows what the market portfolio is, people are advised to buy an index fund, which is hoped to replicate the market portfolio. At the same time, some mutual funds may carbon copy the risk return profile of a portfolio with a particular mix between the market portfolio and the riskfree asset. This may explain why there is a variety of mutual funds alongside every index fund. The problem is that one size may not fit every need. This necessitates the existence of the separate account and hedge fund businesses. There are other reasons why individually tailored portfolios make sense to investors besides competitive advisory fees equivalent to the mutual fund's management charges and sales loads.

In chemistry, there is only one way to produce water, that is, by combining two atoms of hydrogen and one atom of oxygen. There are only two types of atoms used and the technology, or the proportion between the two atoms, is well known. In finance, there is almost an infinite number of ways in which to achieve an identical goal. For example, one can achieve an investor's overall risk target with a variety of securities. They can be categorized as either value or growth stocks or even as emerging market securities. When a portfolio is separately managed privately and if the portfolio performs well, there is little chance that managing the portfolio will ever have any market impact, which will reduce the fund returns. This is the reason why many hedge funds and separate account managers run portfolios in strict confidentiality unlike in the case of publicly traded mutual funds.

However, there are several important issues that must be addressed in creating and managing any efficient portfolio. In this paper, some basic results from Modern Portfolio Theory are borrowed to offer several pedagogies to run optimal portfolios. The paper begins with the concept of the efficient portfolio frontier (EPF), a set of all possible efficient portfolios, of which we will choose the best portfolio for the investor.

Constructing Efficient Portfolio Frontier (EPF)

The efficient portfolio is defined as that portfolio which gives the highest expected return on any given perceived risk. Since, the risk and returns are the *ex ante* quantities the EPF thus constructed must be *ex ante*. Sometimes, people construct *ex post* EPF by estimating the historical sample mean returns and the sample standard deviations, which may not repeat in the future.

To establish the mechanics of generating the EPF, start with an *n* number of candidate stocks, which may qualify for value, growth, large or small cap stocks, or even those that belong to particular sectors. The return on a portfolio r_p with a possible *n* number of stocks is $r_p = x$ 'r where x and r are the column vector of portfolio weights, x_i 's and individual securities returns, r_i 's. Note also that x'l = l, where 1 is a (*n*x1) column vector of 1's. The expected return and the variance of returns on a portfolio are simply

$$E[r_p] = \sum_{i=1}^{n} x_i E[r_i] = \mathbf{x}' E \qquad (Eq. 1)$$

 $\sigma_p^2 = \sum_{i=1}^n x_i^2 \sigma_i^2 + \sum_{\substack{i=1 \ j\neq i}}^n \sum_{j\neq i}^n x_i x_j \sigma_{ij} = \mathbf{x}' \Omega \mathbf{x}$ (Eq. 2)

Matrices E and Ω are a (*n*x1) vector of securities' expected returns and a square variance-covariance matrix of order *n*, respectively. The EPF is generated normally by maximizing the Sharpe ratio, i.e. $\theta = \frac{E[r_p] - r_f}{\sigma_n}$

The EPF is generated normally by maximizing the Sharpe ratio, i.e. $\theta = \frac{-p^2 - f}{\sigma_p}$ with respect to x_i 's subject to the constraint that x'l=1. This is a non-linear programming problem, and if all x_i 's must be non-negative, we use solutions

suggested by Kuhn, *et al* (1951). The optimal value of $x_i = \frac{Z_i}{\sum_{i=1}^{n} z_i}$ is the solution to

an *n* number of simultaneous equations $E - r_f = \Omega z$. The notation *z* is a (*nx*1) column vector of z_i 's, where the solution vector z^* is $z^* = \Omega^{-1} (E - r_f)$. The optimal values of x_i 's are then substituted into $E[r_p]$ and σ_p , which produce one particular point in the EPF. By varying different values of r_f 's, a complete locus of EPF is then generated. Alternatively, one may want to minimize minimizing σ_p instead of maximizing the function θ , but subject to the constraint that the expected return on portfolios must meet certain minimum return requirements. Clearly, parameter structures, i.e. E and Ω , determine the shape of the EPF. Many analysts may have differing views on these parameters.

27

If portfolio returns follow a particular *geometric Brownian* motion, $r_{pt} = \mu_{pt}\Delta t + \sigma_{pt}\varepsilon_{pt}\sqrt{\Delta t}$ where μ_{pt} and σ_{pt} are instantaneous *annual* drift and volatility terms, and ε_{pt} is presumed to be distributed with zero mean and unit variance, then, the forecasting value of portfolios hinges critically upon the value of μ_{pt} and σ_{pt} , that is, a vector of individual securities returns, E, and a variance-covariance matrix, Ω . But then, it requires an economic model.

(1) Multifactor Model

Consider a multifactor model similar in spirit to the one suggested by Professors Fama and French (1992; 1993). Suppose that security returns can be explained by three factors such as the market index return, price-to-book value, and firm size. In order to estimate the sensitivity of securities returns with respect to these independent variables, run a time series regression r = XA + u where r and u are $(T \ge 1)$ column vectors of individual securities returns, r_{ij} 's and regression errors, u_{ij} 's at time t, respectively. X is a matrix of time series regressors. It may also include, as in BARRA (URL: www.barra.com), variables such as industry dummies, price-earnings ratio, price to book, dividend yield, trading activity, 12-month relative strength, logarithm of market capitalization, earnings variability, EPS growth rate, price to sales, debt to equity, price volatility, and the like. Typical assumptions about the property of the model are $E[u_{it}]=0$, $E[u_{it}^2]=v_{it}^2$, $E[u_{it}X_{itt}]=0$, $\forall j \ j = \{1,2,3,4\}$ and $E[u_{it}X_{iht}]=0$ for $j \neq h$. Note that the variance varies with time, i.e. non-constant variance. The appropriate technique to resolve this non-constant variance problem is to use weighted least squares, a special case of a more general econometric technique known as generalized least squares.

The model is greatly simplified if $E[u_{il}u_{lk}]=0$, for $t \neq k$, i.e. no autocorrelation. Existence of a possible heteroscedasticity in the regression error terms means that the heteroscedasticity being as a special case of the generalized least squares, the error structure is $E[uu']=v^2C$, in matrix notation, where

 $C = \begin{pmatrix} 1 & \rho & \rho_2 & \dots & \rho^{T-1} \\ \rho & 1 & \rho & \dots & \rho^{T-1} \\ \rho_2 & \rho & 1 & \dots & \rho^{T-3} \\ \dots & \dots & \dots & \dots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \dots & 1 \end{pmatrix}$

Let us assume that the regression thus estimated is $\hat{r} = X\hat{A}$ for each stock.

The next step of securities valuation is to run another regression but at this time cross sectionally. The regression equation is $y = B\lambda + v$, assuming that there are *n* numbers of stocks. The vector $y = r - r_f$ is the risk premium on individual securities if r_f is the risk-free rate of interest. The remaining matrices are defined as follows:

$$\mathbf{B} = \begin{pmatrix} 1 & a_{11} & a_{12} & a_{13} \\ 1 & a_{21} & a_{22} & a_{23} \\ 1 & a_{31} & a_{32} & a_{33} \\ \vdots & \vdots & \vdots \\ 1 & a_{n1} & a_{n2} & a_{n3} \end{pmatrix} \quad \boldsymbol{\lambda} = \begin{pmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$$

The regression coefficients λ 's typically have special meanings in Arbitrage Pricing Theory, i.e. λ_0 is the Jensen's alpha equivalent, and all other λ 's, the expected risk premium on portfolios sensitive to each one of the original three factors.

Portfolio returns may drift at the rate of μ_{pt} where $\mu_{pt} = \lambda_0 + \lambda_0 a_{p2t-1} + \lambda_3 a_{p3t-1}$ and $a_{pjt-1} = \mathbf{x}^{*}\mathbf{B}_{jt-1} \ \forall j$. The matrix \mathbf{B}_{jt-1} is (*n*x1). The regression error $v_{pt} = y_{pt} - \mu_{pt}$ is assumed to have a zero mean and the volatility equal to σ_{pt} . To generate a series of forecasting values for v_{pt} , it is also necessary to estimate instantaneous volatility σ_{pt} of portfolios returns. Cross sectional regressions can be repeated *T* number of times to compute standard errors for each individual security.

(2) Capital Asset Pricing Model with the Alpha Term

By far the most revealing and yet simple technique is the famous Capital Asset Pricing Model (CAPM). If $y_{it} = r_{it} - r_{ft}$ and $X_{mt} = r_{mt} - r_{ft}$, consider the following time series regression model.

$$y_{it} = \alpha_i + \beta_i X_{mt} + u_{it}$$
(Eq. 3)

The symbol r_{it} is assumed to be log price relatives for a stock *i*, i.e $\lambda n \left(\frac{P_t}{P_{t-N}}\right) \approx r_t$

and similarly, for r_{mt} as well. Sometimes, when r_{it} is regressed on r_{mt} directly without subtracting the risk-free rate, it is called the Single Index Model. If both y_{it} and X_{mt} are random variables assumed to be distributed bivariate normal, Eq. (3) is known as the bivariate regression, and the constant term α_i is the measure of Jensen's excess return on securities.

The stock price P_t is the last price in real time. The time interval N is commonly per day, i.e. N = 1. Eq. (3) assumes by convention $E[u_{it}]=0$, $E[u_{it}^2]=v_{it}^2$ and $E[u_{it}u_{ik}]=0$ for $k \neq t$, $k = \{1,...,T\}$ and $E[\varepsilon_{it}X_{mk}]=0$. Again, this is the regression with non-constant error variance with respect to time. Consequently, the regression technique is to use weighted least squares.

Ignoring the variance on the risk-free rate of interest σ_{pt}^2 for the moment, the expected return, the standard deviation and the covariance of securities returns are then given by

$$E[r_{it}] = \alpha_i + E[r_{ft}] + \beta_i E[X_{mt}]$$
(Eq. 4)

Rhee

Journal of Business and Management - Vol. 12, No. 1, 2006

$$\sigma_{it}^2 \approx \beta_{it}^2 \sigma_{Xt}^2 + v_{it}^2$$
(Eq. 5)
$$\sigma_{ijt} \approx \beta_{it} \beta_{jt} \sigma_{Xt}^2$$
(Eq. 6)

Since the portfolio return is $r_{pt} = \sum_{i=1}^{N} x_i r_{it}$, substituting Eqs. (4), (5) and (6) into Eqs.

(1) and (2) gives the portfolio expected return and the standard deviation of returns as follows:

$$E[r_p] = \sum_{i=1}^{n} x_i \alpha_{it} + E[r_f] + \sum_{i=1}^{n} x_i \beta_{it} E[X_{mt}] = \alpha_{pt} + E[r_f] + \beta_{pt} E[X_{mt}] \quad (Eq. 7)$$

$$\sigma_p = \left[\sum_{i=1}^{n} x_i^2 \beta_{it}^2 \sigma_{Xt}^2 + \sum_{i=1}^{n} \sum_{i=1}^{n} x_i x_j \beta_{it} \beta_{jt} \sigma_{Xt}^2 + \sum_{i=1}^{n} x_i^2 v_{it}^2 \right]^{1/2}$$

$$= \left[\beta_{pt}^2 \sigma_{Xt}^2 + \sum_{i=1}^{n} x_i^2 v_{it}^2 \right]^{1/2} \quad (Eq. 8)$$

Using matrix-vector notation, Eqs. (7) and (8) can be rewritten as:

$$E[r_{pt}] = \mathbf{x}'\mathbf{E} = \mathbf{x}'\alpha + E[r_f] + E[X_{mt}]\mathbf{x}'\beta$$
(Eq. 9)

$$\sigma_{pt}^2 = \sigma_{Xt}^2 \mathbf{x}' (\mathbf{B} + \mathbf{v}) \mathbf{x}$$
(Eq. 10)

The matrices B and v are given by

$$\mathbf{B} = \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \dots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \beta_{23} & \dots & \beta_{2n} \\ \vdots & & & & & \\ \vdots & & & & & \\ \beta_{n1} & \beta_{n2} & \beta_{n3} & \dots & \beta_{nn} \end{pmatrix}$$
(Eq. 11)
$$\mathbf{v} = \begin{pmatrix} \mathbf{v}_1^2 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{v}_2^2 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{v}_3^2 & \dots & \mathbf{0} \\ \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{v}_n^2 \end{pmatrix}$$
(Eq. 12)

30

Elements β_{ii} in the matrix are given by the cross product of betas, i.e. $\beta_{ii} = \beta_i \beta_i$. The optimal portfolio allocation is the solution to the problem that θ is maximized with respect to x_i 's as before subject to x' = 1 and $x_i \ge 0$ $\forall i$ with no short sales assumption. Elton et al (2003) offer some elementary solutions to the market model. Solutions to z_i 's and hence, x_i 's are

$$z_{i} = \frac{\beta_{i}}{\nu_{it}^{2}} \left[\frac{E[r_{i}] - E[r_{f}]}{\beta_{i}} - \sigma_{mt}^{2} \sum_{j=1}^{n} z_{j} \beta_{j} \right] = \frac{\beta_{i}}{\nu_{it}^{2}} \left[\frac{E[r_{i}] - E[r_{f}]}{\beta_{i}} - C^{*} \right] \text{ and}$$

$$C^{*} = \frac{\sigma_{mt}^{2} \sum_{j=1}^{n} \frac{(E[r_{i}] - E[r_{f}])\beta_{j}}{\nu_{jt}^{2}}}{1 + \sigma_{mt}^{2} \sum_{j=1}^{n} \frac{\beta_{j}^{2}}{\nu_{it}^{2}}}$$
(Eq. 13)

The underlying mathematics for EPF has been illustrated in Figure 1. By using CAPM as the basis while allowing for the volatility of the risk-free rate of interest. optimal portfolios are found where the straight line through the expected risk-free rate of interest becomes tangent with the EPF, which we will call the pseudo EPF. In general, the vertical axis would shift to the right as shown if the risk-free rate of interest had been volatile. Furthermore, Eq. (7) and hence Eq. (10) would have had the volatility term for the risk-free rate. Therefore, the pseudo EPF will be shifted to the left by the amount of the volatility of the risk-free rate of interest. In the meantime, Eq. (7) and therefore, Eq. (9) suggests that the pseudo EPF will move up by the expected risk-free rate to generate the true EPF. Note that the whole analysis has been simplified if $\sigma_0^2 = 0$.

Figure 1: Delineating Efficient Portfolio Frontier



Standard Deviation o

Method of Parameter Estimation

The characteristics of EPF are contingent upon the variables as shown in Eqs. (9) and (10). Eqs. (9) and (10) are sensitive to a particular pricing model employed. Therefore, we estimate ex ante values of $E[r_f]$, $E[X_{ml}]$ and σ_{xr}^2 ; and of elements in \hat{A} and v.

(1) Return Generating Process for the Expected Risk-Free Rate

As a first step, consider the three-month U.S. Treasury bill yield and compute the mean and the standard deviation. There are 21 trading days a month or 252 trading days a year. Sample sizes would vary as a function of the fund's investment period. For example, if the fund's investment period were one year, i.e. 252 trading days, we will use 252 data points. Of them, only 251 returns are "historical," and the latest return observation, or the 252nd return data, is computed in real time. Let us assume, as in Vasicek (1977), the risk-free rate of interest follows a *geometric Brownian* motion of the form:

$$dr_f = \lambda (b - r_f) dt + \sigma_f dz$$
 (Eq. 14)

The parameter *b* is the constant long-term mean return, λ is the rate at which the current r_f reverts to the mean and dz is the *Brownian* motion, i.e. $dz = \varepsilon \sqrt{dt}$.

(2) Computing the Time-Dependent Volatility of the Market Risk Premium

Next, we deal with *annualized* daily *log* price relatives for S&P 500 index, from which the historical annual mean return and the standard deviation are also computed. We do not interpolate returns for weekends and holidays. Note that the annual mean return is the daily return multiplied by 252 days and the annual standard deviation is the daily volatility times $\sqrt{252}$. In order to recognize that volatility varies with time, one may consider using the exponentially weighted moving average (EWMA) model, where the latest volatility is assigned a greater weight. The general form of EWMA is, for $0 \le \lambda \le 1$,

$$\sigma_{Xt} = \left[\lambda \sigma_{Xt=1}^{2} + (1 - \lambda) e_{Xt=1}^{2}\right]^{1/2}$$
(Eq. 15)

With EWMA, e_{Xt}^2 over a time period t is often measured by the percent change in the index itself, i.e. $\left(\frac{I_t - I_{t-1}}{I_{t-1}}\right)$. The symbol I_t represents the S&P 500 index at time t.

As an alternative, the GARCH (1, 1) model (Bollerslev, 1986; Engle, et al, 1986) postulates the variance to follow a process,

$$\sigma_{it}^2 = b_3 V + b_1 e_{it-1}^2 + b_2 \sigma_{it-1}^2 + u_{it} = b_0 + b_1 e_{it-1}^2 + b_2 \sigma_{it-1}^2 + u_{it}, \text{ where } \sum_{j=1}^{n} b_j = 1. \text{ Hence},$$

 $b_3 = 1 - b_1 - b_2$. If $b_3 = 0$, $b_1 = 1 - \lambda$, and $b_2 = \lambda$, then GARACH (1, 1) is equivalent to EWMA. The symbol V is a long-run average variance rate. Since $E[\sigma_{it}^2] = \hat{b}_0 + \hat{b}_1 e_{it=1}^2 + \hat{b}_2 \sigma_{it=1}^2$, the variance forecasts can be computed as $E[\sigma_{i,t+k}^2] = V + (\hat{b}_1 + \hat{b}_2)(\sigma_{i,t}^2 - V)$,

Rhee

where $V = \frac{\hat{b}_0}{1 - \hat{b}_1 - \hat{b}_2}$. Similarly, the time-varying covariance between *i* and *j* can be

also written either as the exponentially weighted moving average model, $\sigma_{ij,t}^2 = \lambda \sigma_{ij,t=1}^2 + (1 - \lambda) e_{i,t} e_{j,t}, \text{ or GARCH } (1, 1), \text{ i.e. } \sigma_{ij,t}^2 = b_3 + b_1 e_{i,t} e_{j,t} + b_2 \sigma_{ij,t=1}^2.$

(3) Computing the Expected Market Risk Premium

The market risk premium $X_{mt} = r_{mt} - r_{ft}$ will be assumed to follow the usual return generating process of the form.

$$X_{mt} = \mu_{Xt} dt + \sigma_{Xt} \varepsilon_{Xt} \sqrt{dt}$$
 (Eq. 16)

The symbol μ_{x_t} is the mean return on the market risk premium.

(4) Evaluating the CAPM through Statistical Regression

Suppose that we estimate the regression Eq. (3) for stock *i* as $y = X\hat{A} + e$, where if the sample size is *T*, for each stock *i*

$$\mathbf{y} = \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{y}_r \end{pmatrix} \mathbf{X} = \begin{pmatrix} \mathbf{1} & \mathbf{X}_1 \\ \mathbf{1} & \mathbf{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{1} & \mathbf{X}_T \end{pmatrix} \mathbf{e} = \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{e}_T \end{pmatrix} \hat{\mathbf{A}} = \begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix}$$

It is known that

$A = (X'C^{-1} X)^{-1} X'C^{-1} y$ (Eq. 17)

As for those stocks, which have not been listed for long enough periods, we will use data only to the extent that return figures are available. Consequently, each stock will have as many betas as the total number of investment periods to be considered, as the size of sample varies with the fund's life. Eq. (17) becomes quite simple, if there is only one independent variable just as in the CAPM regression, i.e.

$$\hat{\beta}_{i} = \frac{Sample \ Covariance \ (y_{i}, \ X_{m})}{Sample \ Variance \ (X_{m})} \ \text{and} \ \hat{\alpha} = \overline{y}_{i} - \hat{\beta} \overline{X}_{m}.$$

(5) Estimating the Expected Return on Individual Securities

In order to calibrate the expected return on individual securities, note that the security's pricing equation from the regression is $E[y_{it}] = \hat{\alpha}_i + \hat{\beta}_i E[X_{mt}]$. Since

 $E[y_{it}] = E[r_{it}] - E[r_{ft}]$, the expected return on individual security $E[r_{it}]$ is given by $E[r_{it}] = \hat{\alpha}_i + \hat{\beta}_i E[X_{mt}] + E[r_{ft}]$.

Now, we adjust values for betas, assuming that all securities betas tend to unity. We follow Blume (1975), where the long run predictive beta, $\beta_i^* = \left(\frac{1}{3}\right) + \left(\frac{2}{3}\right) \hat{\beta}_i$.

Consequently, if CAPM holds, the securities pricing equation becomes

$$\hat{E}[r_{it}] = E[r_{ft}] + E[X_{mt}]\beta_i^*$$
 (Eq. 18)

Second, we establish some connectivity between the theoretical expected return as given in Eq. (18) and the securities mean return by assuming that the securities return reverts to the mean, i.e.

$$m_{it} = \hat{E}[r_{it}] + \lambda(\overline{r}_{it} - \hat{E}[r_{it}])$$
(Eq. 19)

Now, we still have to define the instantaneous return volatility and we will assume that to be the standard error of the regression, which is v_i . See Eq. (5). If the forecasting error from the regression is $e_{it} = y_{it} - \overline{y}_{it}$, the variance on y_{it} is conveniently

measured by $v_{it}^2 = \frac{1}{T-1} \sum_{t=1}^{T} e_{it}^2$. We now have described the method of estimating the

securities return generating process as

$$dS_{it} = m_{it}dt + v_{it}\varepsilon_i \sqrt{dt}$$
 (Eq. 20)

Parameter Estimation: An Example

We now show some specific examples in estimating various parameters when constructing an ex ante EPF. We initially consider a randomly chosen 26 stocks in this example. We also assume the fund's life is one calendar year, or 252 trading days.

Step 1: Take *daily log* price relatives for the S&P 500 index and annualize them. Subtract the three-month Treasury bill rates from the annualized S&P index returns to generate a series X_{mt} . Let us assume that the mean and the standard deviation using one-year's worth of sample data are given by:

	Mean	S.D.	
3-month T-bill yield	1.34%	0.02%	
Market risk premium	- 5.45%	26.95%	

Step 2: Estimate the expectations coefficient $0 \le \lambda \le 1$. Simulate Eq. (14), assuming that on a given day, the actual three-month yield r_f was 1.03% with the 1.34% mean

and 0.02% standard deviation. Using $\lambda = 0.10$, b = 1.34% and $\Delta t = \frac{1}{252}$, calibrate the Monte Carlo trajectory. The result is shown in Table 1, where $E[r_f] = 1.07\%$, the last column.

(1)	(2)	(3)	(4)	(5)
	Date	Random Numbers	Growth Rate	Return Forecasting
1	6/7/2003	-0.3002	-0.0000017	0.0103
2	6/8/2003	2.3757	0.0000242	0.0103
3	6/9/2003	0.8746	0.0000097	0.0103
4	6/10/2003	0.0020	0.0000013	0.0103
5	6/11/2003	0.8030	0.0000090	0.0103
	1.0			
			2	
			· · · ·	
248	5/27/2004	1.3151	0.0000139	0.0107
249	5/28/2004	-0.5763	-0.0000043	0.0107
250	6/1/2004	0.4375	0.0000055	0.0107
251	6/2/2004	0.0062	0.0000013	0.0107
252	6/3/2004	0.2326	0.0000035	0.0107

Table 1: Monte Carlo Trajectory for $E[r_{ft}]$

Remarks: Column for Growth is the implementation for Eq. (14), i.e. $dr_f = \lambda (b - r_f) dt + \sigma_f dz$. It has been assumed $\lambda = 0.10$, b = 1.34% and $\Delta t = \frac{1}{252}$.

Forecasted returns at time t are the rate in the preceding period plus the rate given in column (4).

Step 3: Compute the time dependent volatility on the market index return by using Eq. (15). The latest volatility is measured by the percent change in the S&P index. If the annualized percent change in S&P 500 on a particular day is -60.57%, with $\lambda = 0.10$, *ex ante* σ_{xt} is $[0.90 \cdot 0.2695^2 + 0.10 \cdot (-0.6057)^2]^{1/2} = 0.3194$.

Step 4: Conduct a Monte Carlo simulation to generate a series for X_{mt} by using $\mu_{Xt} = -0.0545$ and $\sigma_{Xt} = 0.3194$, i.e. Steps 1 and 3. See Table 2. Based on the result from Monte Carlo simulation trials, we expect the market risk premium will be 14.49% after a period of a year.

Step 5: Compute daily *log* price relatives for each stock. All corporate actions, which may have affected the firm's distribution including, but not necessarily limited to splits, mergers and acquisitions, spin-offs, equity carve-outs, divestitures and the like except cash dividends, will have required return adjustments. However, *ex*-dividend day

returns will not be adjusted. Run a series of regressions for each stock. The resulting regression coefficients and standard errors are reported in Table 3. Table 3 also displays the securities one-year mean return, predictive betas, instantaneous mean returns for all securities per Eq. (18); and the mean reverting drift returns, i.e. Eq. (19).

Step 6: Calibrate the Monte Carlo trajectories for all securities over a one-year period forward by using Eq. (20).

(1)	(2)	(3)	(4)	(5)	(6)
	Date	Random Numbers	Growth Rate	Cumulative Rate	Market Risk Premium Forecasting
1	6/7/2003	0.6817	0.0135	1.0135	0.0135
2	6/8/2003	0.7718	0.0153	1.0290	0.0290
3	6/9/2003	-0.1674	-0.0036	1.0253	0.0253
4	6/10/2003	0.0765	0.0013	1.0267	0.0267
5	6/11/2003	0.7217	0.0143	1.0414	0.0414
	4	4.1		1	1. A.
247	5/26/2004	0.1367	0.0025	1.2344	0.2344
248	5/27/2004	-2.5412	-0.0514	1.1710	0.1710
249	5/28/2004	0.2991	0.0058	1.1778	0.1778
250	6/1/2004	-1.1529	-0.0234	1.1502	0.1502
251	6/2/2004	-0.3708	-0.0077	1.1414	0.1414
252	6/3/2004	0.1646	0.0031	1.1449	0.1449

Table 2: Monte Carlo Trajectory for $E[X_{mt}]$

Remarks: Column (4) the implementation of Eq. (16), i.e. $X_{mt} = \mu_{Xt}dt + \sigma_{Xt}\varepsilon_{Xt}\sqrt{dt}$. It has been assumed that $\mu_{Xt} = -0.0545$ and $\sigma_{Xt} = 0.3194$. Column (5) is given by a formula $\prod (1 + X_{mt})^t$. Column (6) is column (5) minus column one (1).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
		Beta	S.E.	Alpha	Mean	Long Run Beta	Model Hurdle Rate	Drift Return	EXP PR
1	ABC	0.6143	0.3679	-0.0777	-0.1112	0.7429	0.0407	0.0255	0.0775
2	ACV	0.4263	0.1768	-0,0135	-0.0367	0.6175	0.0867	0.0744	-0.1489
3	BLS	1.1724	0.3880	-0.0921	-0.1559	1.1149	0.0802	0.0566	-0.0640
4	CFC	0.9056	0.2668	0.5132	0.4638	0.9371	0.6597	0.6401	0.0817
5	DELL	1.0606	0.2690	0.2136	0.1558	1.0404	0.3750	0.3531	0.0329
6	EIX	0.9473	0.5034	-0.1183	-0.1699	0.9649	0.0323	0.0121	0.0365
7	FBF	1.4209	0.2970	0.0675	-0.0098	1.2806	0.2638	0.2365	0.1139
8	FON	0.6092	0.6240	-0.0166	-0.0498	0.7395	0.1013	0.0862	-0.1924
9	GAS	1.0483	0.5730	-0.0975	-0.1546	1.0322	0.0628	0.0411	0.1128
10	GDW	0.7147	0.1669	0.1939	0.1550	0.8098	0,3220	0.3053	0.0707
11	HAS	0.7079	0.2995	0.2024	0.1639	0.8053	0.3299	0.3133	0.0794
12	HMA	0.5923	0.3297	-0.1330	-0.1652	0.7282	-0.0167	-0.0316	-0.1283
13	IFF	0.5905	0.1946	-0.0689	-0.1010	0.7270	0.0472	0.0324	0.0240
14	IGT	0.8075	0.2428	0.4408	0.3968	0.8717	0.5778	0.5597	0.0654
15	JCI	0.8518	0.1913	0.1009	0.0546	0.9012	0.2423	0.2235	0.0409
16	KSE	0.6202	0.2157	0.0558	0.0220	0.7468	0.1747	0.1594	0.0115
17	LSI	1.8622	0.6453	-0.3843	-0.4857	1.5748	-0.1453	-0.1794	-0.1041
18	MAR	1.0940	0.2647	0.0799	0.0203	1.0627	0.2446	0.2222	0.0514
19	QTRN	0.5753	0.3842	0.0720	0.0407	0.7168	0.1866	0.1720	-0.0322
20	ROH	1.1008	0.2165	-0.0471	-0.1071	1.0672	0.1182	0.0957	0.0209
21	SDS	1.3355	0.4175	-0.0081	-0.0808	1.2237	0.1799	0.1539	0.0359
22	UCL	0.6943	0.2124	-0.1229	-0.1607	0.7962	0.0032	-0.0132	0.0217
23	VMC	0.9536	0.2366	-0.1248	-0.1768	0.9690	0.0263	0.0060	0.0865
24	WWY	0.4853	0.1874	0.0268	0.0003	0.6569	0.1327	0.1194	0.0621
25	XLNX	1.8596	0.5278	0.0088	-0.0924	1.5731	0.2475	0.2135	0.0398
26	ZION	0.9184	0.2179	0.0406	-0.0094	0.9456	0.1884	0.1686	0.0108

Table 3: Parameters for Securities Return

Remarks: The standard error of the regression in Column (4) represents a measure of unsystematic risk. Column (5) is Jensen's excess return. Column (7) is the long-run beta as compared to the historical beta in Column (3). Column (8) is the equilibrium expected return promulgated by the securities pricing model. Column (9) is an implementation of the mean-reverting drift Eq. (19).

The Solution Technique

Before introducing the solution technique more formally, we now offer some intuitive economic explanations about what would be hidden behind complicated mathematical processes. The basic idea has been promulgated in Eq. (13). The idea is quite simple. Consider Figure 2. First, securities are ranked by their risk premium to beta in a descending order. This is represented by a line that declines downward. Next, a series of scenario portfolios will be formed from the portfolio with one best

stock, e.g. WWY in Figure 2, the portfolio of two best stocks, e.g. WWY and HAS, the portfolio of three best stocks, e.g. WWY, HAS, DELL, and so on until adding another stock will actually lower the portfolios risk premium to risk ratio. The portfolios curve will first rise but will eventually start to fall, as shown. The reason is that when the portfolio has only a few stocks, it is subject to volatility risk. Consequently, the portfolios risk premium to risk ratio would be low. As the number of securities to be included in the portfolio increases, the portfolio's standard deviation will fall, which raises the risk premium to risk ratio, but only to a certain extent. The portfolios risk premium to risk ratio will reach a peak eventually before it starts to fall again, as the majority of stocks to be added to the portfolio will have considerably low expected returns. As shown, the optimal portfolio includes only seven stocks. The optimal proportion of stocks in the portfolio is then the proportion that each stock contributes to the portfolios risk premium to risk ratio. And this is represented by the shaded rectangles for each stock, assuming that the entire shaded area under the curve that ranks stocks is 100%.



WWY HAS DELL GM ABC BAX AA

The general solution technique is quadratic programming with inequality constraints

$$\sum x_i = 1; \ 0 \le x_1, \ x_2, \dots, \ x_{26} \le 1$$
(Eq. 21)

We will give several more examples here. No matter what we do, however, we will maximize the objective function given by $\phi = \frac{E[r_p] - E[r_f]}{\sigma_n}$. The final answer depends critically on the vector of individual securities expectations and variance-covariance matrices. Table 4 provides such matrices to perform necessary computations based on the particular numerical examples provided here. See Equations (9) and (10).

The second example is the case in which the proportion of any particular stock should not exceed any more than, let's say, 10%, while still disallowing short sales. Constraints can be written as

$$\sum x_i = 1; x_1, x_2, ..., x_{26} \ge 0; x_1, x_2, ..., x_{26} \le 0.10$$
 (Eq. 22)

The third example is when a person has a stock option and does not wish to sell his or her current shares nor add any more extra shares. Suppose that he or she wishes to maintain a 10% holding in the 5th stock, DELL. With no short sales assumption, then, constraints are

$$\sum x_i = 1; x_1, x_2, \dots, x_{26} \ge 0; x_5 = 0.10$$
 (Eq. 23)

The last two examples will allow short sales. However, we will add extra constraints that no shares of any particular stock can be bought or sold at more than 10% of our initial investment. We will use two separate definitions of short sales, one is conventional, i.e. the fourth example, and the other, Lintner's (1965), i.e. the fifth example. Those two constraints are

$$\sum x_i = 1; x_1, x_2, \dots, x_{26} \ge -0.10; x_1, x_2, \dots, x_{26} \le 0.10$$
 (Eq. 24)

$$\sum |x_i| = 1; x_1, x_2, \dots, x_{26} \ge -0.10; x_1, x_2, \dots, x_{26} \le 0.10$$
 (Eq. 25)

Table 5 presents soutions for each one of these examples.

Probability of Reaching the Target Return and Rebalancing

It should be noted that the above analysis implies certain probabilities with which to achieve the specific investment target. The procedure is quite elementary in that

$$Pr[r_p \ge r_t] = Pr\left[z_p \ge \frac{r_T - E[r_p]}{\sigma_p}\right] = 1 - Pr\left[z_p \le \frac{r_T - E[r_p]}{\sigma_p}\right] \quad (\text{Eq. 26})$$

To give a numerical example, examine Table 4 where the first quadratic programming solution resulted in the portfolio's expected return of 9.18% and the return standard deviation of 29.23%. If the target return on the account is 12% consistent with the investor's risk profiles,

$$Pr\left[z_p \le \frac{r_T - E[r_p]}{\sigma_p}\right] = Pr\left[z_p \le \frac{0.1200 \neq 0.0918}{0.2923}\right] = Pr[z_p \le 0.0965] = 0.5384$$

Consequently, the probability that the portfolio will achieve at least the 12% return is $Pr[r_p \ge 12\%] = 1 - 0.5384 = 46.16\%$, or we are 53.84% certain that the portfolio's

return will not exceed 12%. Rebalancing strategies may depend on whether or not the underlying probabilities and hence, the portfolios expected return and variance change.

System Functionality

Based on the general discussion above, one can visualize the following functionality in designing a comprehensive *portfolio management* system.

(1) Manager Input

Fund life - one, two, three, five, 10 or 20 years

Fund's maximum risk in relation to the market

Fund's absolute target return, as is more conventional for hedge funds

Benchmark index - S&P 500, Wilshire, Russell, Nikkei, FTSE, etc

Investment constraints - Stop loss, liquidity, tolerance level for return computation, upper and lower bounds used for portfolio rebalancing, the maximum amount in percent that a fund can invest in a particular security, the minimum number of shares that must be held of a particular security in a fund, etc.

Rebalancing strategies - aggressive, benchmark, managed, buy and hold, etc

(2) Model Selection

Managers can use the system provided CAPM solutions or are free to build any pricing model of their choosing. For example, in Microsoft Excel, up to 20 explanatory variables can be specified, whether macro or firm specific variables, e.g. foreign exchange rate, unemployment rate, the general interest rate, inflation rate, economic growth, etc. for macro variables; and the price to book, cash flows, sales growth, ROE, inventory turnover, leverage, etc. for firm specific and yet variables in commonality. Explanatory variables can also include technical criteria such as Bollinger Band breakouts, golden cross, RSI, etc.

(3) Stocks under Analysis

These are a group of stocks in the existing portfolio. For a new portfolio, determine investment styles, e.g. momentum, value vs. growth, small vs. large cap, etc. by defining a group of stocks of interest for up to an x number of stocks.

(4) Manager Specific Outputs

Expected returns and volatility risk on each stock are displayed. Also displayed is Value at Risk (VaR) as well as Value to Gain (VtG). The results are manager specific according to the pricing model chosen. Statistics will be provided as to how significant the manager's variables are in explaining securities returns.

(5) Performance Attributes for Existing Portfolios

Past performance is compared to industry performance, just as in BARRA, and all other variables the manager specified in his or her pricing models.

(6) Portfolio Optimization Outputs

By invoking the optimization command, each manager will be given the mathematical result from the portfolio optimization module. Portfolios are optimized to minimize the risk and to maximize the expected return. Note that the result varies for different pricing models chosen.

(7) Performance Monitoring Outputs

Probability to achieve the investment target, various performance measures such as Sharpe, Treynor and Jensen's, Value at Risk for portfolios.

Conclusions

The brokerage industry has long been trying to earn fee income while slowly deemphasizing its traditional commission driven businesses. It has tried the wrap account business and mutual fund allocation models to start offering their customers advisory services. But the industry was not quite able to cross the chasm. With the emergence of the Internet and its wide acceptance as a means of communication, however, the industry is about to burst into two distinct new lines of somewhat nontraditional business. They are the online advisory based planning business and the separately managed account, all of which target high net worth markets. The hard reality is that, although many are enthralled by the convenience of the online advisory solutions especially in the 401(k) retirement market, the role of financial advisors is not quite clear. Unless the brokerage industry simply licenses the software to end users with brokerage commissions embedded in the license fees, the only viable alternative is to pursue the separately managed market.

For the online advisory business, the industry offers a variety of fund products. A single or multiple funds are offered as a part of advisory tools online. Separately managed accounts are different in that portfolios are particularly custom tailored according to the client's needs, which underlies the whole premise of the business. The separately managed account not only offers good theories but also makes sound business sense.

This paper has shown that, in principle, the best asset allocation model should be based on *ex ante* and not *ex post* efficient portfolio frontiers. Specific solutions to the portfolio selection problem are contingent upon the particular nature of variancecovariance matrix of securities returns. This results from particular pricing models used in the model. Desires to discover the sources of "alpha" can be easily accommodated by allowing portfolio managers to try out their own "factors" in the model. In other words, managers should be allowed to assess their own portfolio frontiers. That is, make all portfolios truly unique to all managers and to all investors.

	- 1. Sec	Example 1	Example 2	Example 3	Example 4	Example 5
1	ABC	12.43%	10.00%	12.16%	10.00%	2.82%
2	ACV	0.00%	0.00%	0.00%	-10.00%	-10.00%
3	BLS	0.00%	0.00%	0.00%	-10.00%	-5.14%
4	CFC	6.02%	10.00%	5.07%	10.00%	4.83%
5	DELL	0.00%	0.00%	10.00%	10.00%	-0.94%
6	EIX	0.00%	0.00%	0.00%	0.03%	0.04%
7	FBF	9.31%	10.00%	11.53%	10.00%	5.13%
8	FON	0.00%	0.00%	0.00%	-10.00%	-3.97%
9	GAS	8.75%	10.00%	9.63%	10.00%	1.61%
10	GDW	13.68%	10.00%	8.18%	10.00%	10.00%
11	HAS	14.90%	10.00%	14.46%	10.00%	4.16%
12	HMA	0.00%	0.00%	0.00%	-10.00%	-10.00%
13	IFF	0.00%	0.00%	0.00%	10.00%	0.32%
14	IGT	0.00%	10.00%	0.00%	10.00%	4.28%
15	JCI	0.00%	9.08%	0.00%	10.00%	1.91%
16	KSE	0.00%	0.00%	0.00%	10.00%	-2.52%
17	LSI	0.00%	0.00%	0.00%	-10.00%	-2.91%
18	MAR	0.00%	0.92%	0.00%	10.00%	1.26%
19	QTRN	0.00%	0.00%	0.00%	-1.13%	-2.80%
20	ROH	0.00%	0.00%	0.00%	2.62%	-3.44%
21	SDS	0.00%	0.00%	0.00%	-1.52%	-0.63%
22	UCL	0.00%	0.00%	0.00%	10.00%	-1.39%
23	VMC	11.81%	10.00%	11.47%	10.00%	6.56%
24	WWY	23.11%	10.00%	17.51%	10.00%	8.62%
25	XLNX	0.00%	0.00%	0.00%	-10.00%	-0.72%
26	ZION	0.00%	0.00%	0.00%	10.00%	-4.00%
		100.00%	100.00%	100.00%	100.00%	100.00%
Portfolios Expected Return		0.0918	0.0899	0.0896	0.1652	0.0799
Portfolios Standard Deviation		0.2923	0.3038	0.3053	0.3025	0.0697
Max	imized Sharpe	0.2774	0.2607	0.2584	0.5107	0.9926

References

- T. Bollerslev. (1986). Generalized Autoregressive Conditional Heteroscedasticity. Journal of Econometrics, 31, 307-27.
- Blume, M. (1975). Betas and their regression tendencies. Journal of Finance, 3, 785-795.
- E. Elton, M. J. Gruber, S. J. Brown & W. N. Goetzmann. (2003) Modern Portfolio Theory and Investment Analysis (6th ed.) John Wiley & Sons, Inc.
- R. F. Engle and T. Bollerslev. (1986). Modeling the persistence of conditional variances. *Econometric Reviews*, 5.1, 1-50.
- Fama, E. & K. French, (1993). Common Factors in the Returns on Bonds and Stocks. Working Paper, Center for Research in Security Prices, University of Chicago. , (1992) The cross section of expected stock returns. Journal of

Finance 47, no. 2, June: 427-466.

- J. Lintner (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *Review of Economics and Statistics*, XLVII, February: 13-37.
- H.W. Kuhn & Tucker, A.W. (1951). Nonlinear Programming, in J. Neyman (ed.), Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, University of California Press, Berkeley and Los Angeles
- O.A. Vasicek (1977). An Equilibrium Characterization of the Term Structure. Journal of Financial Economics, 5, 177-88.