

# A Hybrid Location Method for Missile Security Team Positioning

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*This research provides solutions to the problem of locating security teams over a geographic area to maintain security for US Air Force Intercontinental Ballistic Missile Systems. A combination of two location modeling techniques, the  $p$ -median and  $p$ -center models, is used to generate solutions which minimize the distances traveled while minimizing the maximum distance any one missile site is from required security forces. Solutions are generated using heuristic and optimization techniques in a VBA enhanced Excel spreadsheet. Results indicate that a significant improvement can be achieved and the techniques are currently being tested by the Air Force for possible implementation.*

The Minuteman Intercontinental Ballistic Missile (ICBM) nuclear weapon system has been a pillar of the United States' military forces for more than forty years and will continue to be so for the foreseeable future. The current version of this weapon system, the Minuteman III, is organized into operational units called "wings" at three locations: Malmstrom Air Force Base (AFB), Montana—200 ICBMs; Minot AFB, North Dakota—150 ICBMs; and F. E. Warren AFB, Wyoming—150 ICBMs. All wings are broken down into squadrons of 50 ICBMs each and flights of 10 ICBMs. A site containing an ICBM is known as a Launch facility (LF), and in the Minuteman system an LF contains only one ICBM. A Missile Alert Facility (MAF) is also assigned to each flight. The MAF houses the launch control officers, flight security controller, and additional support personnel.

The Minuteman weapon system has earned credibility as a viable strategic deterrent through its ability to achieve high availability levels on a consistent basis, normally exceeding 99 percent annually. Availability is commonly defined as the measure to which a system is in an operable state at the start of a mission when the mission is called for at an unknown random time (Blanchard & Peterson, 1995). Maintenance personnel performing priority, periodic, and weapon system upgrade maintenance are a key aspect of achieving these availability rates. Additionally, protecting the weapon system from damage, destruction, and theft is crucial to the nation's security. Therefore, a specified number of security teams are positioned to respond to a threat at an LF. These security teams only need to be positioned when the LF is to be penetrated, that is, when the maintenance team will enter into the missile silo itself. Additional requirements ensure that additional forces are also close enough to respond if a hostile event escalates.

The events of September 11, 2001 have placed a much higher degree of emphasis on security for ICBMs. The United States cannot afford the dire consequences of damage or theft of even one of its nuclear assets. Over the course of the past several years, many physical security upgrades have been implemented. These physical security upgrades, along with the existing system, are designed to delay a hostile act long enough for a security force to respond to any threat and eliminate it. Great measures are taken to protect the system as it lies in its standby state, and great measures must also be taken to protect it when it is exposed for maintenance. Recent demands from the highest levels of government to reduce security forces' response times place an increased strain on the already tenuous availability of limited security forces resources. Therefore, effectively deploying available security forces and exercising sound decision-making policies when completing all daily maintenance requirements is the only way to ensure system protection and effectiveness.

These enhanced security requirements require a balance between achieving system availability and affording the proper level of protection to the weapon system. It is unlikely that both goals can be maximized on a consistent basis without some trade-offs occurring. Therefore, decision makers are put in the tenuous position of having the maintenance schedule constrained by security requirements on a daily basis. Constant cancellation of maintenance actions will undoubtedly cause system availability to degrade over time, while even more dire consequences are perceivable without adequate security for the weapon system. This research seeks to provide decision makers with a reliable method for balancing these competing objectives.

The overall research question for this study is: Can a user-friendly modeling technique be developed to minimize security force response times while providing decision makers with a tool for balancing trade-offs between maintenance requirements and optimal or near optimal security forces' response times? This question addresses the current operating environment and acknowledges the possibility of requiring some trade-offs in system availability to achieve heightened security levels.

## Literature Review

Very little previous research has been conducted on this specific problem. An initial effort by Seaberg (1999) introduced the umbrella concept. This concept was to create a security “umbrella” under which maintenance would be conducted. This proposal attempted to limit maintenance operations that required a penetrated LF to remain within the umbrella. The proposal also had a goal of limiting missile maintenance operations to daylight hours. In related work, James (2004) proposed the option of removing critical nuclear components from the missile and performing all annually required maintenance during a set period of time. This method would require extensive coordination between the various types of maintenance teams and would result in much overtime pay for civilian workers. This method purports to reduce the number of security teams required per day, but comes at a high cost. Although several classified exercises and studies have intended to analyze the inherent vulnerabilities of the system and potential physical security preventative measures, no known studies have been conducted concerning the optimal placement of security teams in the missile fields. This study accomplishes such placement with the use of existing location modeling techniques found in the literature (Drezner & Hamacher, 2002; Daskin, 1995).

## Methodology

This study uses actual data on LF and MAF locations, interconnecting roads in the missile fields, and available security forces’ teams. These data are applied in three distinct methods to understand and study the tradeoffs between maintenance and security requirements. Each of these methods can be described as a discrete location model composed of different types of servicing locations and demand locations. The servicing facilities are the LFs, MAFs, and selected staging areas located at road intersections. Demand nodes are the penetrated LFs where maintenance is being accomplished. The models aspire to optimize the geographic placement of security force response teams based on daily scheduled maintenance requirements. Although classified procedures related to this work could not be included in this paper, the military’s approach to this research and related problems are referenced in Eberlan (2004) and are contained in the classified security regulation, Department of Defense S-5210.41-M.

The next several sections provide a description of the data, the formulation of the models, methods for generating solutions, and the necessary modeling assumptions.

## Data

This research utilizes data collected on the 150 Minuteman III LFs at F.E. Warren AFB, Wyoming for calendar year 2003 and from January through May of 2004. The following data was collected from the F.E. Warren AFB during that period:

- Security forces’ response time matrices from Missile Alert Facilities (MAFs) to LFs

- Daily maintenance status sheets from January through May of 2004
- Periodic maintenance due dates and locations
- Daily security escort availability and security teams requested by maintenance
- Daily LF maintenance performed
- Off-alert hours for missile systems

All data were collected from historical records maintained at FE. Warren AFB and were obtained from the Headquarters of Twentieth Air Force (Overholts, 2004). A limited amount of weekend and holiday data was missing on security escort availability and number of teams requested by maintenance. Data on LF and MAF latitude/longitude coordinates were collected from a public website (<http://w3.uwyo.edu/~jimkirk/warren-mm.html>) and coordinates for the additional staging areas were obtained by using the free trial-version of AccuGlobe®. Road overlays for Colorado, Nebraska, and Wyoming were also obtained at a public website (<http://arcdata.esri.com/data/>), and the latitude and longitude coordinates of the staging areas were obtained by plotting the staging areas on these maps and viewing the displayed coordinates. Once generated, all of the coordinates were validated with personnel at FE. Warren AFB.

Candidate locations for security team positioning included the 165 existing Minuteman LFs and MAFs, and 68 additional staging sites were selected at road intersections resulting in 233 candidate locations. Coordinates of candidate locations were represented in degrees, minutes, and seconds format and distances were calculated using the Haversine great circle method outlined by Bell & McMullen (2003). This distance data was then used to build a distance matrix detailing the mileage between each LF, MAF, and staging area as computed by an Excel® macro developed by Eberlan (Eberlan, 2004). For this research, straight-line distances were used and it is acknowledged that using actual road route distances would improve the accuracy of the results, but would also add greatly to the complexity of the problems to be solved. Finally, distance calculations were converted to a response time in minutes by multiplying the distance by a factor of 60/40, to represent forty miles per hour average driving speeds.

## Model Formulations

The problem of finding the optimal placement for security force teams is modeled as a facility location problem. Three of the many methods of locating facilities available in the literature are selected to solve this problem. This gives decision makers alternate methods to solve the problem based on their objectives. First, The p-median problem intends to minimize the demand-weighted total distance between demand sites and servicing facilities (Hakimi, 1964). The p-center problem covers all demand and seeks to minimize the maximum distance between any single demand point and a servicing facility (Hakimi, 1964). In addition, a third hybrid method is developed by first obtaining a p-center solution, and then adjusting the solution by reducing the total distance using a p-median approach.

### The $p$ -Median Problem

The formulation of Daskin (1995) with three minor adjustments is utilized in this research. The first adjustment removes the demand weight multiplier from the objective function, because the demand in this model is assumed equal. This is consistent with the demand at FE. Warren where each penetrated LF is assumed equal in importance to any other penetrated LF. The second adjustment allows for fewer than the available number of security teams to be utilized. This is necessary when the number of available security teams exceeds the number of scheduled sites or when deploying additional security teams will not improve upon the objective function. The final adjustment allows each penetrated LF to be assigned to more than one security team. This is feasible at FE. Warren because, theoretically, security teams may be placed in close enough proximity to one another to allow for an overlap of coverage. That is, one team could respond to an LF within another team's assigned coverage area in the rare event that the other team is already responding at another LF. The formulation is as follows:

$$\text{MINIMIZE} \quad \sum_i \sum_j d_{ij} Y_{ij} \quad (1)$$

$$\text{SUBJECT TO:} \quad \sum_j Y_{ij} = 1 \quad \forall i \quad (2)$$

$$\sum_j X_j \leq P \quad (3)$$

$$Y_{ij} - X_j \leq 0 \quad \forall i, j \quad (4)$$

$$X_j \in \{0,1\} \quad \forall j \quad (5)$$

$$Y_{ij} \in \{0,1\} \quad \forall i, j \quad (6)$$

WHERE:

$X_j = 1$  if we locate a security team at candidate staging area  $j$ , 0 otherwise

$Y_{ij} = 1$  if penetrated LF  $i$  is served by candidate staging area  $j$ , 0 otherwise

$d_{ij}$  = the distance between points or nodes  $i$  and  $j$

$P$  = number of security teams to be located.

The objective function (1) minimizes aggregate travel distance, thus minimizing response times, between all penetrated LFs and selected staging areas where security teams are placed. Constraint (2) requires that at least one team covers each penetrated LF. Constraint (3) states that no more than  $P$  teams are to be located. Constraint (4) links the location variables ( $X_j$ ) and the allocation variables ( $Y_{ij}$ ), and ensures that a penetrated LF,  $i$ , cannot be assigned to a candidate staging area,  $j$ , unless a team is located at staging area  $j$ . Constraints (5) and (6) are integrality constraints. The

GRASP heuristic is used to generate solutions for the p-median problem (Feo & Resende, 1989). The heuristic begins by checking all possible combinations of scheduled LFs as potential staging areas and also searches the areas around those points. The best solution found, which has the minimum total distance, is kept. The randomized portion of the heuristic is then performed, evaluating the neighborhoods around 100 randomly chosen points and comparing the solutions to the best original solution. If a better solution is found, it is kept as the very best solution. The solution identifies the locations of the staging areas, the allocations of penetrated LFs to staging areas, and the response distance/time. This model assumes all teams are available.

### *The p-Center Problem*

The objective of the p-center model is to minimize the maximum response time or distance between a security team placed at a staging area and a penetrated LF. There are two different formulations of the p-center problem: the vertex p-center problem and the absolute p-center problem. The vertex p-center formulation is used in this model because staging areas can only be located on the candidate staging area nodes and not on the arcs (anywhere along the routes), as in the absolute p-center problem. The formulation used in this research is from Daskin (1995) with minor adjustments. As in the previous modeling techniques used in this chapter, this modeling formulation again removes the demand-weighted multiplier. The same adjustments pertaining to security teams made in the p-median formulation are included in this model.

MINIMIZE  $W$

$$\text{SUBJECT TO: } \sum_j Y_{ij} = 1 \quad \forall i \quad (7)$$

$$\sum_j X_j \leq P \quad (8)$$

$$Y_{ij} - X_j \leq 0 \quad \forall i, j \quad (9)$$

$$\sum_j d_{ij} Y_{ij} \leq W \quad \forall i \quad (10)$$

$$X_j \in \{0,1\} \quad \forall j \quad (11)$$

$$Y_{ij} \in \{0,1\} \quad \forall i, j \quad (12)$$

WHERE:

$W$  = maximum distance between a penetrated LF and its assigned team

$Y_{ij}$  = 1 if penetrated LF  $i$  is assigned to candidate staging area  $j$ , 0 otherwise

$X_j = 1$  if we locate a team at candidate staging area  $j$ , 0 otherwise

$P =$  number of security teams to locate

$d_{ij} =$  the distance between points or nodes  $i$  and  $j$

The objective function (7) minimizes the maximum distance that any penetrated LF is from a deployed security team. Constraint (8) requires that each penetrated LF be assigned to at least one security team. Constraint (9) stipulates that no more than  $P$  teams are to be located. Constraint (10) links the location variables ( $X_j$ ) and the allocation variables ( $Y_{ij}$ ), and ensures that a penetrated LF,  $i$ , cannot be assigned to a candidate staging area,  $j$ , unless a team is located at staging area  $j$ . Constraint (11) states that the maximum distance between a penetrated LF and a security team must be greater than or equal to the distance between any penetrated LF,  $i$ , and the team at staging area,  $j$ , to which it is assigned. Constraints (12) and (13) are the integrality and non-negativity constraints, respectively. This model is solved to optimality by using the Bisection method to achieve the lowest maximum distance any team is located from a penetrated LF. Because the maximum distance between any two points in the distance matrix is 93.29 miles, the method begins with a maximum value of forty-seven and a minimum value of zero. The maximum and minimum values are bisected until the lowest distance that covers all scheduled penetrated LFs, within one-tenth of a mile, is found. The Bisection method can be slow to converge to the optimal solution, but is guaranteed to obtain the optimal solution within the specified accuracy (Faires & Burden, 1993).

## Hybrid Method

The hybrid method is a heuristic approach which utilizes the p-center and p-median formulations and solution methods previously described. This is the only method that seeks to achieve multiple objectives, and the heuristic method contains three steps. First, the Bisection method is used to solve for the p-center solution. Second, the maximum distance from the resulting p-center solution is then fixed. In the third and final step, the GRASP heuristic is employed to minimize the total distance given the p-center solution maximum distance constraint. The resulting hybrid solution is simply a heuristic adjustment to the optimal p-center solution which achieves a nice balance or compromise between the competing objectives of minimizing distance (p-median) and minimizing the maximum distance (p-center). The hybrid method uses the same distance computation methods used for the first two models and is used to generate solutions that are then compared to the others in the results section. The hybrid method is subject to the same modeling assumptions as the p-median and p-center techniques.

### Modeling Assumptions

Several critical assumptions are made in developing the models:

- No consideration is given to higher emergency security levels
- The straight line distance computations used in the models are realistic
- Data collected from FE. Warren AFB is representative of the other wings
- Maintenance requirements for other missile systems were not considered

- The given number of Security Teams is always available
- A security team responding to a penetrated LF is unavailable to respond to another penetrated LF

## Results

The current method of deploying security teams in the Air Force is based solely on experience and the daily requirements in the missile fields. Since there is no established mathematical method associated with the current deployment scheme, the results of the three models are compared to each other for analysis. A comparison of the generated results to the actual response times at FE Warren AFB is not releasable from the US Air Force; however, a limited comparison of using mathematical modeling in comparison to actual response times is reported by Overholts (2006). For this research, each model was used to generate simulated results for maintenance requirements which occurred during a period of 53 days from January 2004 to May 2004. This period was felt to be an adequate representation of the maintenance encountered at FE. Warren, and a copy of the generated results data from the model is presented in Appendices A-C. In summary, each model is compared to the others based on several measures including security team utilization, average security team response time, average total distance to penetrated LFs, and the average maximum distance any security team is located from a penetrated LF. Additionally, the models are run with all potential staging areas and with a limited set including only the MAF and LF locations as potential sites for positioning security teams. This comparison was useful to show the Air Force that restricting its staging areas to only the MAF and LF locations is having a negative impact on their ability to cover penetrated LFs. The MAF/LF only option includes 165 potential staging areas, and the all staging area options include an additional 68 sites for a total of 233 potential staging areas located throughout the missile field. The combined results are shown in Table 1 and the daily results are listed in Appendices A-C.

**Table 1:** Comparison of Model Mean Results

	All Staging Areas			
	Usage	Response time	Total Distance	Max Distance
<b>p-median</b>	97.84%	4.92 mins	28.83 miles	10.95 miles
<b>p-center</b>	97.84%	7.73 mins	43.94 miles	7.30 miles
<b>Hybrid</b>	97.84%	5.76 mins	33.75 miles	7.30 miles
	MAF/LF Only Staging Areas			
<b>p-median</b>	97.84%	12.38 mins	59.93 miles	12.98 miles
<b>p-center</b>	90.52%	13.44 mins	66.55 miles	12.53 miles
<b>Hybrid</b>	97.84%	12.71 mins	62.31 miles	12.53 miles



The results are consistent with the objectives of the three models. Also, since five security teams were available for the time period modeled, the p-value for each daily model run ranged from 3 to 5 as shown in Appendices A-C. For all staging areas, the mean total distance from security team locations to penetrated missile sites is the smallest in the p-median model at 28.83 miles, and the mean maximum distance that any team is located from a penetrated site is minimized by the p-center model to 7.30 miles. The Hybrid model makes an additional contribution by fixing this maximum distance at 7.30 miles and by adjusting the teams to accomplish a mean total distance of 33.75 miles. This model nicely balances the objectives of the two models and reduces the average response time to 5.76 minutes for all teams while ensuring that no single demand location is too far from a security team. Similar results are seen in the reduced MAF/LAF only data set. This time, the hybrid model is able to minimize the maximum security team distance to 12.53 miles using the p-center approach and then is able to reduce the overall mean response time to 12.71 minutes. Again, a balance between the two objectives is achieved.

Furthermore, testing using paired t-tests reveals that the mean results in Table 1 are statistically significant for the 53 days of testing depending on the method used. In the all staging areas data set, the p-median method (mean = 4.92) is able to achieve a significantly lower response time in comparison to the p-center method ( $t = -8.39$ ,  $p < .001$ ,  $df = 52$ ) and the hybrid method ( $t = -5.87$ ,  $p < .001$ ,  $df = 52$ ). Significant mean differences are also achieved for mean total distance, and the p-median achieves a significantly lower value in comparison to the p-center ( $t = -8.26$ ,  $p < .001$ ,  $df = 52$ ) and hybrid method ( $t = -5.38$ ,  $p < .001$ ,  $df = 52$ ). The maximum distance to a serviced site is significantly higher for the p-median in comparison to the p-center ( $t = 8.34$ ,  $p < .001$ ,  $df = 52$ ) and the hybrid method is able to achieve a smaller value in comparison to the p-median ( $t = -8.35$ ,  $p < .001$ ,  $df = 52$ ). However, the only insignificant difference for the nine possible tests in the all staging areas data set was for the comparison of the maximum distance for the hybrid and p-center methods ( $t = 1.40$ ,  $p < .165$ ,  $df = 52$ ). This again highlights the ability of the hybrid method to achieve lower response times while not compromising the maximum distance to any one serviced site. Finally, similar results were found for the MAF/LF only data set where the same eight out of nine paired t-tests for the models were also found to be statistically significant.

## Conclusions

The results of this research clearly indicate the need for a more analytical approach to positioning security teams to meet missile security requirements. Although the Air Force goes to great lengths to determine and enforce security requirements, it is clear that efficiencies can still be realized by properly positioning the limited number of security teams available. In addition, the differences seen between the use of three different approaches appears to indicate that there are clear tradeoffs in time and distance based on which overall objectives are selected by military commanders. This research gives commanders a choice between three different methods with different objectives and shows how security times and distances will differ depending on

objectives. Although this research uses data from F.E. Warren Air Force Base in Wyoming, it is believed that the techniques used here are also generalizable to similar missile bases in Montana and North Dakota. This is especially true due to identical constraints, policies and procedures used at these locations. The missiles at these locations are also organized similarly and are dispersed over a large geographic area. Up until now, research to validate these procedures at bases in Montana and North Dakota has not been undertaken by the Air Force, but is planned for future research. Additionally, it can be seen that the potential set of staging sites makes a significant difference in response times. If commanders limit staging areas to only secure MAF/LF locations, they would do so at the expense of doubling average response times and distances in the example studied in this research. Finally, continuing research on this subject is aimed at developing a model which simultaneously schedules maintenance jobs while positioning security teams in order to maximize the overall availability of a group of missiles over a defined time period. This effort will not only provide a tool for commanders in the field, but will also help them study the tradeoff between the amounts of maintenance which can be accomplished while meeting the hard constraint of security.

This research has served as a demonstration of how to apply a modeling technique to an operational security problem to get a better solution rather than relying on unplanned experience. We are not saying that these are the only or best ways to solve the problem, rather the p-center and p-median methods are techniques easily understood and applied using the standard software packages already available to security personnel in the Air Force. This is also beneficial since the cost to implement these techniques is minimal, since the software to apply them is already available. Additionally, the bases have access to Operations Research professionals who have the skills to maintain the models once implemented, thereby reducing costs.

Finally, we contend that non-military business operations face similar decision making for limited resources, especially for operational security coverage. For example, recent research in Dallas by Ma (2006) has used similar methods to position police officers. Additionally, extensive research for public services such as schools, libraries, health care, and pharmacies have benefited from adaptations of the p-median, p-center and other covering models as presented by Marianov and Serra (2002). Also, other location analysis methods such as the capacitated facility location problem (Canel et al, 2001) were not selected here, but may be desirable in a situation where the total costs of the final location decision are considered. Additionally, it should also be recognized that other methods such as hierarchical techniques (Marianov & Serra, 2001) may also be used to further adapt the p-center and p-median models to the specific problem attributes for this and other research; and that expanded formulations of these location models are also described in the literature (Dekle et al, 2005). Future research should also consider the use of heuristic solution techniques such as Heuristic Concentration (Rosing & Reville, 1997) which has a two-phase approach similar to the Hybrid method, or artificial intelligence techniques such as Genetic Algorithms (Bozkaya, Zhang & Erkhut, 2002) or Tabu Search (Ohemuller, 1997) which have been found to be effective for location analysis problems.

Overall, it is believed that problems faced by military organizations in this paper

are quite similar to resource allocation and location coverage problems faced in civilian industries and services. We believe the lessons and methods used in this paper apply not only to the military to protect the nation's nuclear arsenal, but apply equally to managers who face the task of protecting valuable or sensitive assets with similarly constrained security personnel and budgets.

*The views expressed in this article are those of the authors and do not reflect the official policy or position of the Air Force, Department of Defense or the U.S. government.*

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## Appendix A p-Median Results

Date	Number of Teams Available	Scheduled Sites	P-Median								
			All Staging Areas				MAF/LF Only				
			Average Response Time (Minutes)	Number of Teams Used	Total Distance	Maximum Distance of any Team to a Penetrated LF	LFs not covered	Average Response Time (Minutes)	Number of Teams Used	Total Distance	Maximum Distance of any Team to a Penetrated LF
10/2004	5	E-3,E-8,E-10,G-10,J-8,K-8,K-11	392	5	1828	986	1329	5	5737	1050	
18/2004	3	E-3,E-8,E-10,F-6,G-10,K-11,N-5	922	3	4305	1396	1436	3	8704	1764	
17/22004	5	D-11,E-2,G-10,J-4,K-2,K-5,K-7,K-9,K-11	486	5	2919	934	1078	5	6469	1233	
17/32004	5	B-2,E-5,E-8,G-10,J-4,K-2,K-7,K-11	462	5	2463	1381	1178	5	6283	1233	
17/42004	5	B-2,E-5,E-8,G-10,K-11,K-2,K-7	232	5	1083	587	1126	5	5251	966	
1/202004	5	A-3,A-8,B-3,D-6,D-7,E-9,G-10,J-6,K-4,K-9,K-11	953	5	6962	1582	1508	5	11058	1934	
12/12004	5	A-3,A-8,C-6,D-6,E-7,E-9,E-10,G-2,G-10,K-9,K-11	750	5	5502	1520	1162	5	8519	1177	
12/22004	4	A-3,A-8,B-3,C-9,G-10	327	4	1089	1089	1330	4	4433	1015	
12/72004	4	A-11,A-8,D-2,D-6,E-10,E-7,G-2,G-8,10,K-5	1166	4	7773	2062	1414	4	9427	2279	
1/282004	4	A-2,A-8,A-11,E-7,E-10,G-2,H-9	630	4	2941	1390	1138	4	5312	1050	
1/292004	5	E-7,G-2,A-2,H-9,A-8,A-11	116	5	464	464	1065	5	4262	1015	
2/23004	5	D-8,E-11,G-2,E-2,A-8,K-11,D-4,H-7,C-5,D-7	712	5	4746	1342	1308	5	8723	1215	
2/42004	5	D-8,E-11,G-2,E-10,E-2,A-8,K-11,D-4,C-5,D-7	669	5	4463	1342	1266	5	8374	1112	
2/52004	5	D-8,E-11,G-2,C-4,A-2,A-8,A-10,C-5,D-9,E-2,G-11	579	5	4248	1020	1116	5	8181	1112	
2/92004	4	E-11,G-2,C-5,A-5,O-11,E-9	361	4	1444	979	1119	4	4475	1471	
2/102004	5	E-11,B-4,D-5,D-7,D-9,D-10,D-11,E-4,G-11,M-4	529	5	3529	1223	1104	5	7361	1077	
2/112004	5	J-10,E-11,G-11,M-4,A-5,A-10,A-4,J-6,E-9,D-10,F-9	654	5	4795	1297	1292	5	9178	1271	
2/122004	5	E-11,D-7,B-4,D-10	000	4	000	000	1067	4	2845	1005	
2/172004	4	A-8,B-4,D-11,E-11,E-6,E-9,F-4,G-11,J-8,K-7,K-8,N-11,N-7	1380	4	11936	2866	1605	4	13906	2534	
2/182004	4	G-11,E-9,A-6,B-4,C-6,D-11,E-6,E-11,F-4,F-11,J-6,K-7,K-8,N-11	1060	4	12696	2621	1628	4	15180	1668	
2/192004	4	G-11,E-9,H-2,A-4,A-6,A-8,B-4,C-6,E-11,F-4	923	4	6154	1528	1430	4	8533	2343	
2/232004	4	E-9,G-3,A-9,A-11,K-10,N-9	764	4	3096	2101	1413	4	5651	2158	
2/242004	4	E-9,G-3,K-10,A-11,F-4,A-7,A-9,B-7,D-3	964	4	5785	1749	1480	4	8881	1297	
2/252004	4	E-9,G-3,A-7,A-8,E-7,O-9,L-4,A-5	749	4	3689	1499	1498	4	7992	1327	
2/262004	4	E-9,G-3,D-7,L-10,E-8	138	4	461	461	1117	4	3124	1045	
3/22004	4	A-9,G-3,J-3,N-7,E-2,K-11,E-6	1194	4	5574	2311	1607	4	7499	2054	
3/32004	3	G-3,E-6,E-7,D-4,E-10,E-11	686	3	3202	1050	1036	3	4939	1050	
3/42004	5	G-3,E-6,F-11,D-4,E-10,E-11,K-11	274	5	1278	729	1068	5	4965	1050	
3/82004	4	E-6,G-4,A-8,B-7	000	4	000	000	1174	4	3130	1015	
3/92004	4	E-6,G-4,H-9,B-7,A-2,A-8,D-4,H-10	617	4	3292	1724	1206	4	6432	1184	
3/102004	4	E-6,G-4,H-9,A-2,A-8,D-4,H-10	586	4	2732	1724	1126	4	5249	1015	
3/112004	4	A-9,E-6,G-4,H-9,A-8	141	4	471	471	1016	4	3396	1015	
3/152004	4	D-5,G-4,A-5,E-8	000	4	000	000	1271	4	3398	982	
3/162004	3	G-4,E-8	000	2	000	000	1184	2	1579	807	
3/172004	4	G-4,E-8,A-8,G-6	000	4	000	000	1332	4	3552	1015	
3/182004	5	G-4,E-6,A-5,A-8	000	4	000	000	1278	4	3408	1015	
3/222004	4	E-8,E-8,G-9,A-5,B-4	276	4	918	918	1078	4	3593	982	
3/232004	5	E-8,A-5,G-9,G-11,E-8	000	5	000	000	1236	5	4121	1012	
3/242004	5	E-8,E-8,A-8,H-8,C-4	000	5	000	000	1093	5	3543	874	
3/252004	5	E-8,H-8,A-6	000	3	000	000	1141	3	2282	874	
3/292004	4	E-8,E-10,M-5,M-6	000	4	000	000	1078	4	2876	1050	
3/302004	4	E-8,E-10,M-5,M-6	000	4	000	000	1078	4	2876	1050	
4/52004	4	E-10,B-4,M-9,J-11,N-6,G-9	896	4	3582	1837	1332	4	5410	1423	
4/62004	5	D-9,E-10,B-6,B-7,B-9,F-4	150	5	601	601	1424	5	5696	1637	
4/72004	5	E-10,N-6,B-9,B-10,B-11,F-4,G-6,M-5	559	5	2682	1971	1442	5	7692	1637	
4/122004	3	E-10,A-8,A-11,B-7,L-10,M-5,C-3	1054	3	4819	1815	1410	3	6579	1730	
4/142004	3	H-11,B-4,B-5,A-11,A-8,B-6	636	3	2542	1513	976	3	3912	1015	
4/292004	5	O-4,M-9,C-3,A-9,J-5,O-9	257	5	1026	1026	983	5	3933	916	
5/32004	5	G-7,O-4,C-3,M-8,A-7,B-11,E-11,L-2,L-11	867	5	5202	2111	1443	5	8659	2007	
5/42004	5	O-4,G-7,M-8,B-11,L-2,L-11,A-7,B-6,C-9,N-9,O-9	1007	5	7384	2400	1404	5	10290	1900	
5/52004	5	O-4,G-7,M-8,B-6,C-9,N-9	265	5	1682	1082	1218	5	4872	1205	
5/62004	5	O-4,G-7,A-11,C-8,G-2	000	5	000	000	943	5	3143	885	
5/102004	4	O-4,G-7,A-11,C-8,G-2	412	4	1372	1372	943	4	3143	885	
Total	232	Averages and Total number of Teams Used	492	227	2883	1096	1238	227	5993	1298	
			% used	97.94%			% used	97.94%			
			Max.	Max.	Max.		Max.	Max.	Max.		
			1380	12696	2866		1628	15180	2534		
			Min.	Min.	Min.		Min.	Min.	Min.		
			000	000	000		943	1579	807		





